Entropy of the FRW cosmology based on the brick wall method

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Abstract

The brick wall method in calculations of the entropy of black holes can be applied to the FRW cosmology in order to study the statistical entropy. An appropriate cutoff satisfying the covariant entropy bound can be chosen so that the entropy has a definite bound. Among the entropy for each of cosmological eras, the vacuum energy-dominated era turns out to give the maximal entropy which is in fact compatible with assumptions from the brick wall method.

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The holographic principle in quantum gravity suggested by 't Hooft [1] shows that degrees of freedom of a spatial region reside on its boundary. In the thermodynamics of black holes, it has been known that the entropy of a black hole is proportional to the area of the horizon [2, 3]. Recently, the holographic principle has been studied in the cosmological context with a particle horizon [4]. In the cosmology, the choice of holographic boundary is in some sense unclear; in contrast to the black hole physics, for which there is a definite boundary called event horizon. However, it has been shown that the thermodynamic first law $dU = TdS + WdV$ can be satisfied if the boundary is chosen to be the cosmological apparent horizon, where U, S, V, T, and W are the internal energy, the entropy, the volume, the temperature, and the work density, respectively [5, 6, 7, 8]. This result has been induced from Einstein field equations when the temperature is defined by $T = |\kappa|/2\pi$ with the entropy $S = \mathcal{A}/4G$, where κ, G, and $\mathcal{A}$ are the surface gravity, the gravitational constant, and the area of the apparent horizon, respectively.

On the other hand, there are various methods to obtain the entropy proportional to the area of the event horizon of a black hole. One of the most convenient way to get the statistical entropy is to use the brick-wall method [9], where the cutoff parameter is introduced because of divergence near the event horizon, and it has been applied to various black holes [10, 11, 12, 13, 14, 15]. Since degrees of freedom of a field are dominant near horizon, the brick-wall model has often been replaced by a thin-layer model, which makes the calculation of entropy simple [16, 17]. Recently, it has been shown that the (thin-layered) brick-wall model can be applied to a time-dependent black hole with an assumption of local equilibrium near horizon [18].

Now, we would like to consider the entropy of Friedmann-Robertson-Walker (FRW) universe based on the brick-wall model, since there exists the apparent horizon satisfying the thermodynamic first law. Moreover, this is the first application of this method to this cosmological model as far as we know. For this purpose, the apparent horizon of the universe will be introduced as the holographic boundary instead of the event horizons of black holes, and the entropy of the FRW universe will be formulated by means of the brick-wall method without specific solutions of the scale factor without loss of generality. Then, the entropy is calculated for the various eras of the universe, and it is explicitly shown that the entropy can never exceed the gravitational entropy with the same boundary, $S \leq S_g = \mathcal{A}/4G$, which is supported by the covariant entropy bound(CEB) [19].

Let us now start with the standard FRW metric,

$$ds^2 = -dt^2 + a^2(t)\left[\frac{dr^2}{1-kr^2} + r^2 d\Omega^2\right], \tag{1}$$

where $k = 0, \pm 1$ are normalized spatial curvatures, and $d\Omega^2 = d\theta^2 + \sin^2\theta d\phi^2$ is the line element of unit two-sphere. To get the statistical entropy by the brick wall method [9], we change the radial coordinate to $R = ra$ for convenience, then the metric (1) can be written in the form of

$$ds^2 = -\left(\frac{1 - R^2/R_A^2}{1 - kR^2/a^2}\right)dt^2 - \frac{2HR}{1 - kR^2/a^2}dtdR + \frac{dR^2}{1 - kR^2/a^2} + R^2 d\Omega^2, \tag{2}$$

where $H = \dot{a}/a$ is the Hubble parameter, and the apparent horizon R_A is given by

$$R_A = \frac{1}{\sqrt{H^2 + k/a^2}}. \tag{3}$$

Then, the Klein-Gordon equation $[\Box - \mu^2]\Psi = 0$ in this background is explicitly written as

$$\left[\partial_t \frac{-\partial_t - HR\partial_R}{\sqrt{1 - kR^2/a^2}} - \frac{1}{R^2}\partial_R \frac{HR\partial_t - (1 - R^2/R_A^2)\partial_R}{R^{-2}\sqrt{1 - kR^2/a^2}} - \frac{\mu^2 + R^{-2}\ell(\ell+1)}{\sqrt{1 - kR^2/a^2}}\right]\psi(t,R) = 0, \tag{4}$$

and separation of variables is possible up to $\Psi = \psi(t,R)Y_{\ell m}(\theta,\phi)$, whereas $\psi(t,R)$ is no longer separable. Note that the FRW cosmology also obeys the holographic principle [4, 5, 6], and it is sufficient to investigate fields in the vicinity of horizon in order to get the statistical entropy. Now, we assume the frequency to be constant near horizon as like time-dependent Vaidya metric in Ref. [18], so that the field becomes $\psi(t,R) = e^{-i\omega t + iv(t,R)}$. Using the WKB approximation, $R \sim R_A \gg 1$ along with slowly varying condition, $\dot{v} \ll \omega$, the radial momentum can be defined by

$$v' \simeq \frac{-HR\omega \pm \sqrt{(1 - kR^2/a^2)\omega^2 - (1 - R^2/R_A^2)[\mu^2 + R^{-2}\ell(\ell+1)]}}{1 - R^2/R_A^2} \equiv k_R^{\pm}, \tag{5}$$

where the plus(minus) sign represents the out(in)-going wave. Note that the phase velocity of wave is $v_p = \omega/k_R^{\pm}$ in this approximation.

Actually, the present nonstatic cosmological model should be regarded as a nonequilibrium system. However, the notion of local equilibrium as a working hypothesis may be used in the nonequilibrium thermodynamics [20]. So, we will assume that the temperature of thermal radiations is slowly varying near the apparent horizon. In fact, the temperature at the apparent horizon is approximately proportional to the inverse of the apparent horizon,

$T \sim R_A^{-1}$, which will be explicitly calculated in later, and then the local equilibrium requires that

$$\frac{\delta T}{T} \sim \frac{\delta R_A}{R_A} \sim \frac{\delta a}{a} \ll 1, \tag{6}$$

where δ represents fluctuation of each quantity, and the inequality is satisfied as far as the Hubble parameter is very small, $H \ll 1$.

Next, the number of radial modes according to the semiclassical quantization rule is given by

$$\begin{aligned} 2\pi n &= \int_{R_A-h-\delta}^{R_A-h} dR\, k_R^+ + \int_{R_A-h}^{R_A-h-\delta} dR\, k_R^- \\ &= 2\int_{R_A-h-\delta}^{R_A-h} dR \frac{\sqrt{(1-kR^2/a^2)\omega^2 - (1-R^2/R_A^2)[\mu^2 + R^{-2}\ell(\ell+1)]}}{1-R^2/R_A^2}, \end{aligned} \tag{7}$$

where h and δ are cutoffs, and both of them are assumed to be very small positive quantity compared to the apparent horizon, $h, \delta \ll R_A$. Then, the total number of modes for given energy ω is given by

$$N = \int d\ell (2\ell+1) n, \tag{8}$$

where the integration goes over those values for which the square root in the radial momentum (5) is real. Following 't Hooft [9], the free energy is given by

$$\begin{aligned} \beta F &= \int dN \ln\left(1 - e^{-\beta\omega}\right) \\ &= -\int d\omega \frac{\beta N}{e^{\beta\omega} - 1} \\ &= -\frac{\beta}{\pi}\int_{R_A-h-\delta}^{R_A-h} dR \left(1 - \frac{R^2}{R_A^2}\right)^{-1} \int d\omega \left(e^{\beta\omega} - 1\right)^{-1} \int d\ell(2\ell+1) \\ &\qquad \times\sqrt{(1-kR^2/a^2)\omega^2 - (1-R^2/R_A^2)[\mu^2 + R^{-2}\ell(\ell+1)]}. \end{aligned} \tag{9}$$

Now, in the approximation of $\mu^2 \ll R_A/\beta^2\epsilon$, $\epsilon \ll 1/H^2 R_A$, where $\epsilon = h, h+\delta$, the free energy in the leading order is simplified as

$$F \simeq -\frac{\pi^3}{90}(HR_A)^3 \left(\frac{R_A}{\beta}\right)^4 \frac{\delta}{h(h+\delta)}, \tag{10}$$

and then the main contribution of the apparent horizon to the internal energy U and the entropy S are given by

$$U = \frac{\partial}{\partial\beta}(\beta F) \simeq \frac{\pi^3}{30}(HR_A)^3 \left(\frac{R_A}{\beta}\right)^4 \frac{\delta}{h(h+\delta)}, \tag{11}$$

$$S = \beta(U - F) \simeq \frac{2\pi^3}{45}\left(\frac{HR_A^2}{\beta}\right)^3 \frac{R_A\delta}{h(h+\delta)}, \tag{12}$$

respectively, where we used the unified first law, $dU = TdS + WdV$. Since proper lengths of cutoffs h and δ are given by $\bar{h} = \int_{R_A-h}^{R_A} dR/\sqrt{1-R^2/R_A^2} \simeq \sqrt{2R_A h}$, $\bar{\delta} = \int_{R_A-h-\delta}^{R_A-h} dR/\sqrt{1-R^2/R_A^2} \simeq \sqrt{2R_A(h+\delta)} - \sqrt{2R_A h}$, the entropy (12) reads

$$S \simeq \frac{\pi^2}{45}\left(\frac{HR_A^2}{\beta}\right)^3 \frac{\mathcal{A}}{\bar{h}^2} \tag{13}$$

for an arbitrary $\bar{\delta}$ satisfying $\bar{h} \ll \bar{\delta} \ll R_A$, where $\mathcal{A} = 4\pi R_A^2$ is the area of the apparent horizon which depends on time.

Next, we calculate thermodynamic quantities for some cosmological eras. For this purpose, let us consider the equations of motion for the FRW cosmology,

$$H^2 + \frac{k}{a^2} = \frac{8\pi G}{3}\rho + \frac{1}{3}\Lambda, \tag{14}$$

$$\dot{H} - \frac{k}{a^2} = -4\pi G(\rho + p), \tag{15}$$

where ρ is the energy density, p is the pressure, and Λ is the cosmological constant. They can be recast in the form of

$$H^2 = \frac{8\pi G}{3}(\rho_{\rm tot} + \rho_k), \tag{16}$$

$$\dot{H} + H^2 = \frac{4\pi G}{3}(1+3\gamma)\rho_{\rm tot}, \tag{17}$$

where $\rho_{\rm tot} = \rho + \rho_\Lambda = \rho_m + \rho_{\rm rad} + \rho_\Lambda$, $\rho_\Lambda = \Lambda/8\pi G$, $\rho_k = -3k/8\pi G a^2$. The equation-of-state parameter $\gamma = p/\rho$ for each type of energy is listed in table I for convenience; in particular, γ in Eq. (17) is defined by

$$\gamma = \frac{p_{\rm tot}}{\rho_{\rm tot}} = \frac{\Omega_{\rm rad}/3 - \Omega_\Lambda}{\Omega_{\rm tot}}, \tag{18}$$

where $p_{\rm tot} = p + p_\Lambda = p_m + p_{\rm rad} + p_\Lambda$. The density parameter for a certain type of energy ρ_i ($i = m, {\rm rad}, \Lambda$, etc.) was defined by $\Omega_i = \rho_i/\rho_c$, and $\rho_c = 3H^2/8\pi G = \rho_{\rm tot} + \rho_k$ is the critical density. Here, the subscript 'tot' means the sum of actual energy density and pressure, excluding the contributions from the spatial curvature [21].

Now, the inverse of the temperature of a system on the apparent horizon is defined by $T = \beta^{-1} = |\kappa|/2\pi$, where $\kappa = (2\sqrt{-h})^{-1}\partial_a\sqrt{-h}h^{ab}\partial_b R|_{R=R_A}$ is the surface gravity at the apparent horizon, h_{ab} is defined by $ds^2 = h_{ab}dx^a dx^b + R^2(x)d\Omega^2$, and $a, b = 0, 1$ [22]. Then, the temperature for the FRW cosmology is calculated as

$$\begin{aligned} \beta^{-1} &= \frac{H^2 R_A}{2\pi}\left|1 + \frac{1}{2}\left(\frac{\dot{H}}{H^2} + \frac{k}{H^2 a^2}\right)\right| \\ &= \frac{1}{2\pi R_A}\left|\frac{1-3\gamma}{4}\right|, \end{aligned} \tag{19}$$

	rad	m	k	Λ	p-law
$\rho \sim$	a^{-4}	a^{-3}	a^{-2}	a^{0}	a^{-n}
$\gamma =$	$1/3$	0	$-1/3$	-1	$n/3-1$

TABLE I: The energy density ρ and equation-of-state parameter $\gamma = p/\rho$ are calculated for radiation-, matter(m)-, spatial curvature(k)-, and vacuum energy(Λ)-dominated universes. In general, for an energy density evolving as power-law, $\rho \sim a^{-n}$, the equation-of-state parameter is given by $\gamma = n/3 - 1$ [21].

where we used the equations of motion (16) and (17). The apparent horizon (3) in Eq. (19) was written in the form of

$$R_A = \frac{1}{H\sqrt{1-\Omega_k}} = \frac{1}{H\sqrt{\Omega_{\rm tot}}}. \tag{20}$$

From now on, we have to assume $\Omega_k < 1$ so that the apparent horizon is well-defined and the total energy density is positive, $\Omega_{\rm tot} = 1 - \Omega_k > 0$. Then, the internal energy (11) and the entropy (13) are rewritten as

$$U = \frac{R_A}{240\pi\Omega_{\rm tot}^{3/2}\bar{h}^2}\left|\frac{1-3\gamma}{4}\right|^4 = \frac{3\mathcal{F}^{4/3}}{4}\mathcal{M}, \tag{21}$$

$$S = \frac{\mathcal{A}}{360\pi\Omega_{\rm tot}^{3/2}\bar{h}^2}\left|\frac{1-3\gamma}{4}\right|^3 = \mathcal{F}\frac{\mathcal{A}}{4G}, \tag{22}$$

when we set the cutoff as

$$\bar{h} = \frac{\ell_P}{\sqrt{90\pi}\Omega_{\rm tot}^{3/4}}, \tag{23}$$

where $G = c^3\ell_P^2/\hbar$, ℓ_P is the Planck length, $\mathcal{M} = R_A/2G$ is the Misner-Sharp energy [23], and the factor $\mathcal{F}$ is given by

$$\mathcal{F} = \left(\frac{|1-3\gamma|}{4}\right)^3. \tag{24}$$

Note that the cutoff (23) may depend on time, since the density parameter $\Omega_{\rm tot} = 1 - \Omega_k = 1 + k/H^2a^2$ can be time-dependent, for instance, for the case of $k = \pm 1$. In fact, it is due to the nonstatic apparent horizon as discussed in Ref. [18]. However, since the recent observational data [24] leads to the density parameter $\Omega_{\rm tot} \simeq 1$ for the present universe, the cutoff may be time-independent in the approximation of the leading. More specifically, it has been widely accepted that recent observations show that $\Omega_{\rm tot} \simeq 1$, $\Omega_m \simeq 0.3$, and $\Omega_\Lambda \simeq 0.7$ for the present universe, which means $|\Omega_k| \ll 1$, that is, $|\rho_{k0}| \ll \rho_{m0}, \rho_{\Lambda 0}$, where

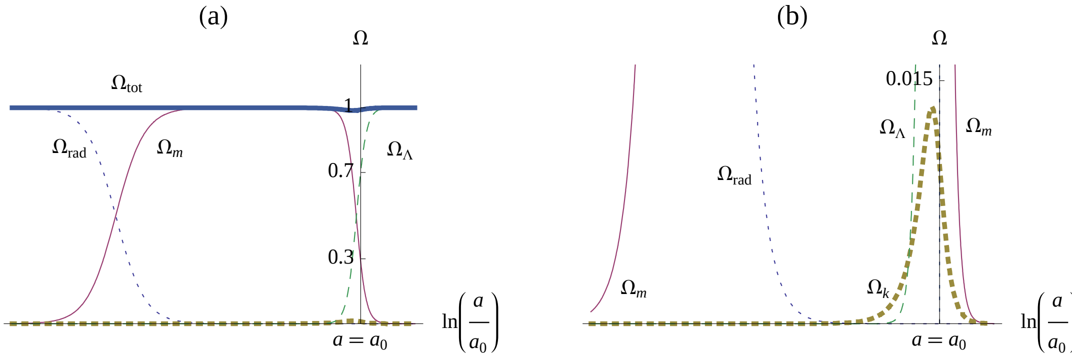

FIG. 1: The density parameters $\Omega_{\rm tot}$(thick, solid line), $\Omega_{\rm rad}$(thin, dotted line), Ω_m(thin, solid line), Ω_Λ(dashed line), and Ω_k(thick, dotted line) are shown with respect to the logarithmic scale, where a_0 represents the present scale of the universe. The profiles are plotted for $k=-1$, but they are not so much different from the case of $k=1$; simply speaking, Ω_k changes its sign and $\Omega_{\rm tot}$ becomes slightly large than 1 near the present scale a_0. (a) It shows that $\Omega_{\rm tot}\simeq 1$. (b) In order to see the detail profile of Ω_k, we magnify the bottom of (a) with the same scale in the horizontal axis.

the subscript 0 represents the values estimated for the present universe scale a_0. Moreover, as shown in table I, $|\rho_k|\sim a^{-2}$, $\rho_m\sim a^{-3}$, and $\rho_\Lambda\sim a^0$. Then, for the past universe, i.e. $a<a_0$, $|\rho_k|=|\rho_{k0}|(a_0/a)^2\ll\rho_{m0}(a_0/a)^3=\rho_m$, and for the future universe, i.e. $a>a_0$, $|\rho_k|=|\rho_{k0}|(a_0/a)^2\ll\rho_{\Lambda0}(a_0/a)^0=\rho_\Lambda$. Thus, we get $|\rho_k|\ll\rho_m+\rho_\Lambda\lesssim\rho_{\rm tot}$ for the whole history of the universe; in other words, $|\Omega_k|\ll\Omega_{\rm tot}$, which yields $\Omega_{\rm tot}\simeq1$ and $|\Omega_k|\ll1$ for all eras of the universe, since $\Omega_{\rm tot}+\Omega_k=1$. The density parameters with respect to the scale factor in logarithmic scale are simply plotted in Fig. 1. As a result, the cutoff (23) can time-independent in the leading order,

$$\bar{h}\simeq\frac{\ell_P}{\sqrt{90\pi}}. \tag{25}$$

It is interesting to note that there has been a conjecture that the entropy of a system within a certain boundary is less than or equal to the gravitational entropy, $S\le S_g=\mathcal{A}/4G$, which is referred to as the CEB [19], and the entropy (22) with the cutoff (23) meets the CEB, since the factor (24) is in the range of $0\le\mathcal{F}\le1$ due to the fact that the equation-of-state parameter (18) is restricted to $-1\le\gamma\le1/3$. If $\gamma<-1$, then the entropy (22) seems to be incompatible with the CEB because of $\mathcal{F}>1$. This is because the CEB is valid only when the source energy satisfies the dominant energy condition which is violated for the energy of $\gamma<-1$. On the other hand, our result for the closed universe($k=1$)

satisfies the CEB, which seems to be different from the previous works that the entropy is not bounded for the closed universe [5]. The essential reason why the present entropy (22) can be bounded even for $k = 1$ is that we have considered only the model describing the late-time accelerated expansion without the big crunch which plays a key role to the unboundedness of the entropy in the closed universe.

In what follows, we would like to investigate the statistical entropy when each energy is dominant. First of all, for the radiation-dominated universe, $\Omega_{\rm rad} \gg \Omega_m + \Omega_\Lambda$, that is $\gamma \simeq 1/3$, the temperature (19) is very low and $\mathcal{F} \ll 1$ so that the internal energy (21) and the entropy (22) are very small. However, it is the early stage of the universe so that the scale factor might not be sufficiently large in applying the present WKB approximation. Moreover, the behavior of the vanishing entropy looks similar to that of the extremal black hole so that we have to use other methods to get desired results. Second, for the matter-dominated universe, $\Omega_m \gg \Omega_{\rm rad} + \Omega_\Lambda$ and $|\gamma| \ll 1$, the entropy (22) is written as

$$S \simeq \frac{\mathcal{A}}{4^4 G}, \tag{26}$$

and the internal energy (21) is given by $U \simeq 3\mathcal{M}/4^5$. Finally, we consider the vacuum energy-dominated universe, $\Omega_\Lambda \gg \Omega_{\rm rad} + \Omega_m$ and $\gamma \simeq -1$, then the entropy (22) is rewritten as

$$S \simeq \frac{\mathcal{A}}{4G}, \tag{27}$$

and the internal energy (21) is given by $U \simeq 3\mathcal{M}/4$. In general, considering power-law cosmology, $\rho_{\rm tot} \sim a^{-n}$ and $\gamma \simeq n/3 - 1$, the entropy (22) can be written as

$$S = \mathcal{F}\frac{\mathcal{A}}{4G}, \tag{28}$$

and the internal energy (21) is given by $U \simeq 3\mathcal{F}^{4/3}\mathcal{M}/4$, where $\mathcal{F} = |1 - n/4|^3$. Note that the power-law inflation model, $n \ll 1$ leads to the same entropy with that of the vacuum energy-dominated universe.

In conclusion, motivated by the holographic principle on the cosmology [4], we have studied the entropy of the FRW cosmology in terms of the brick wall method. As a result, we have shown that the entropy is proportional to the area of the apparent horizon and it obeys the modified area law, $S = \mathcal{F}\mathcal{A}/4G$, and satisfies the CEB, $S \leq \mathcal{A}/4G$, as long as we set the cutoff as Eq. (25). Especially, it is interesting to note that the maximally saturated entropy satisfying the exact one-quarter of the area appears at the vacuum energy-dominated

era, which is related to the most promising candidate of the recently suggested cosmological model to describe the second accelerated expansion of the universe.

The final comment to be mentioned is that the Misner-Sharp energy can be written as $\mathcal{M} = R_A/2G = \rho_{\rm tot} \times V_0$, where $V_0 = 4\pi R_A^3/3$ is the spatially flat($k = 0$) volume. The volume factor V_0 in the Misner-Sharp energy does not change for the $k = \pm 1$ case, which seems somewhat awkward because the volume for the case of $k = \pm 1$ is given by $V_\pm \simeq \pi R_A^3$ in the limit of $|\Omega_k| \ll 1$, and the Misner-Sharp energy can not be written in the form of $\mathcal{M} = \rho_{\rm tot} \times V_\pm$ for the $k = \pm 1$ case. However, as commented in Ref. [5], the difference is due to the contribution of the spatial curvature to the energy. In addition, if we consider the real matter contribution as $M = \rho_{\rm tot} \times V_\pm$, then the internal energy (21) becomes $U = M$ for $\mathcal{F} = 1$ in the case of $k = \pm 1$ and $|\Omega_k| \ll 1$.

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